# Stable cooperation emerges in stochastic multiplicative growth

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Understanding the evolutionary stability of cooperation is a central problem in biology, sociology, and economics. There exist only a few known mechanisms that guarantee the existence of cooperation and its robustness to cheating. Here, we introduce a new mechanism for the emergence of cooperation in the presence of fluctuations. We consider agents whose wealth change stochastically in a multiplicative fashion. Each agent can share part of her wealth as public good, which is equally distributed among all the agents. We show than, when agents operate with long time-horizons, cooperation produce an advantage at the individual level, as it effectively screens agents from the deleterious effect of environmental fluctuations.

Introduction.— The emergence and the stability of cooperation is a central problem in biology, sociology, and economics [1-5]. Cooperation produces an advantage for the group, through the creation and sharing of social goods, but is inherently unstable to cheating and to the tragedy of the commons, where individual agents benefit from the social good without contributing to its creation [6, 7]. The dilemma of the evolution of cooperation can be solved in presence of one or more specific mechanisms [8], which lead to the emergence and long-term stability of the cooperative trait.

Many systems of interest for the study of cooperation exist in a context subject to fluctuation and stochasticity. A paradigmatic model for these systems, which has applications in both economics and population biology, is geometric Brownian motion, which describes the stochastic dynamics of a variable x(t) as  $\dot{x} = \mu x + \sigma x \xi(t)$ , where  $\xi(t)$  is a delta-correlated white noise. In biology, x could represent the abundance of a population, in economics x is the value of an asset or the wealth accumulated by a gambler. In the following we will refer to x as wealth or value of an agent, keeping however in mind the breadth of the applications of the geometric Brownian motion. In general, this equation describes the diffusion limit of growth under a stochastic multiplicative process (see Supplemental Material (SM) [9] section S1). An essential feature of multiplicative growth is that it lacks ergodicity [10], as the time-average behavior differs from the ensemble average. The latter grows exponentially in time with rate  $\mu$ , while the former grows with rate  $g = \lim_{t\to\infty} \langle \log x(t) \rangle / t = \mu - \sigma^2 / 2$ . This difference parallels the difference between arithmetic mean (which corresponds to the ensemble average) and geometric mean (which converges to the time average), and it is the deep reason why the latter is a natural quantity to optimize for agents aiming at maximizing their future profits or growth. In the context of gambling, the Kelly criterion defines the optimal size of a bet based on optimization of the geometric mean [11]. In evolutionary biology, under varying environmental conditions, natural selection favors traits on the basis of their geometric mean fitness [12, 13]. An important consequence of the fact that the geometric mean fitness determines the optimal solution is that not only the average environment but also the amplitude of its fluctuations determine its value, as the geometric average grows with rate  $\mu - \sigma^2/2$ . Reducing fluctuations, i.e., reducing the value of  $\sigma$ , has, therefore, a positive effect and should be expected to be advantaged by natural selection [14].

In the context of growth under fluctuating conditions, we introduce the possibility of cooperation between G agents, by generalizing the setting of Refs. [15–17]. We formulate the model in the context of time-discrete multiplicative random processes. At each time-step, the wealth of each agent changes stochastically as it is multiplied by a random variable. Each agent can also share a fraction  $a_i \in [0, 1]$  of her wealth  $x_i(t)$  Agents can share a fraction of their value as a public good, which is then divided equally among the agents. If none of the agents share anything  $(a_i = 0$  for all i) the agents' value growth are uncoupled, and in the diffusion limit, the model reduces to a system of uncoupled geometric Brownian motion equations. The presence of sharing  $(a_i > 0)$  couples the dynamics of agents' value. In this context it is useful to define that an agent, in a time step  $\Delta t$ , shares a fraction  $a_i = \alpha_i \Delta t$  of her wealth. The parameter  $\alpha_i$  represents therefore a sharing rate, i.e., the fraction of value shared per unit time. For  $\Delta t \to 0$ , we

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obtain the diffusion limit (see SM [9] section S2)

$$\dot{x}_{i}(t) = \mu x_{i}(t) + \sigma x_{i}(t)\xi_{i}(t) + \frac{1}{G}\sum_{j\neq i} \left(\alpha_{j}x_{j}(t) - \alpha_{i}x_{i}(t)\right) , \qquad (1)$$

where  $\xi_i(t)$  are delta-correlated white noises. We will first consider the case where agents experience independent fluctuations, i.e., the noises  $\xi_i$  and  $\xi_j$  are uncorrelated.

The full defector scenario  $\alpha = 0$  corresponds to the original Geometric Brownian motion solution  $g_i = \mu - \sigma^2/2$ . If all the agents fully cooperate  $(\alpha_i \to \infty \text{ for all } i)$ , one can obtain an exact solution of the trajectories  $x_i(t)$  [15–17] resulting in an higher growth rate  $g_i = \mu - \sigma^2/(2G)$ . The intuition behind these results is that, in this context, cooperation produces an advantage as it reduces effectively variability. By sharing their values with others, agents effectively diversify their investments, making their values less subject to fluctuations and, therefore, leading to faster growth. Sharing with more agents further reduces the effect of stochasticity, producing a positive relationship between long-term growth and group size G. This result shed the light on the importance of cooperation under fluctuating conditions: cooperation screens individuals from the negative effect of variability. However, it does not explain how cooperation produces an individual advantage over defection. Also in the simple context of the prisoner dilemma, cooperation produces an individual advantage over defection, when all agents cooperate (i.e., cooperation is Pareto optimal). The dilemma is, as well known, that cooperation is not stable (given that all the other agents are cooperating is advantageous for the individual to defect) while defection is (if all the agents are defecting there is no advantage in starting cooperating).

In this Letter, we explore the stability and origin of cooperation under fluctuating conditions, using the setting of Eq. (1). We show that the maximization of the individual long-time return leads to the emergence and stability of cooperation. We further explore the robustness of these results to correlated fluctuations, colored noise, costly cooperation, and finiteness of time-horizons. We show that, for large enough time-horizons, arbitrary levels of correlated fluctuations, noise time-correlation and costly sharing, cooperation (either full or partial) is advantageous at the individual level. Finally, we explore the effect of these results on an explicit evolutionary dynamics.

Emergence and stability of cooperation.— In order to make analytical progress on Eq. (1) it is convenient to introduce  $q_i(t) := \ln(x_i(t))$ . The quantity that agents optimise is simply  $g_i = \lim_{t\to\infty} \langle q_i(t) \rangle / t$ . The dynamics of  $q_i$  can be obtained from Eq. (1) using Itô calculus. In the case of two agents (G = 2) one obtains

$$\langle \dot{q}_1 \rangle = \mu - \frac{\sigma^2}{2} - \frac{\alpha_1}{2} + \frac{\alpha_2}{2} \langle \exp(q_2 - q_1) \rangle(t)$$
 (2)

In the case  $\alpha_i = 0$  one recovers  $g_i = \mu - \sigma^2/2$ . In every other case, the growth rate of the geometric mean of agent i, in presence of another agent with resource sharing ratio  $\alpha_j$ , will therefore depend on both  $\alpha_i$  and  $\alpha_j$  and will be denoted it by  $g_{\alpha_i|\alpha_j}$ . In the simple case of two agents, we can treat  $g_{\alpha_i|\alpha_j}$  as the payoff function of a continuous game. We aim then at finding the (pure-strategy) Nash equilibria and the evolutionary stable strategies.

It is possible to show (see SM [9] section S3) that the dynamics of  $\exp(q_2 - q_1)$  — the only non trivial term in Eq. (2) — is ergodic with a stationary distribution, leading to a well defined term  $\langle \exp(q_2 - q_1) \rangle_{eq}$ . The growth rate  $g_{\alpha_i | \alpha_j}$  will be equal to  $g_{0|0} + (\alpha_j \langle \exp(q_2 - q_1) \rangle_{eq} - \alpha_j)/2$ .

We also obtain the analytic expression of the stationary distribution of  $(q_2 - q_1)$ , from which we are able to calculate explicitly  $\langle \exp(q_2 - q_1) \rangle_{eq}$  and therefore the long-term growth rate. In particular the effect of cooperation can be quantified by the difference

$$g_{\alpha_1|\alpha_2} - g_{0|0} = -\frac{\alpha_1}{2} + \frac{\sqrt{\alpha_1\alpha_2}}{2} \frac{K_{-1+\frac{\alpha_2-\alpha_1}{2\sigma^2}}\left(\frac{\sqrt{\alpha_1\alpha_2}}{\sigma^2}\right)}{K_{\frac{\alpha_2-\alpha_1}{2\sigma^2}}\left(\frac{\sqrt{\alpha_1\alpha_2}}{\sigma^2}\right)} ,$$
(3)

where  $K_{\beta}(z)$  is the modified Bessel function of the second type. Fig. 1 shows that our analytical solution of Eq. (3) correctly matches the numerical simulations. For fixed strategies  $\alpha_1$  and  $\alpha_2$ , the effect of cooperation increases monotonically with  $\sigma$ : the higher are the fluctuations, the higher is the advantage of cooperation. Interestingly however, for a fixed value of  $\alpha_2$ , the long-term growth rate is not monotonic in  $\alpha_1$ .

The relevant question then reads: given a strategy of the second player  $\alpha_2$ , what is the optimal value of  $\alpha_1$ ? Mathematically, what is the value of  $\alpha^*(\alpha_2) := \operatorname{argmax}_{\alpha} g_{\alpha|\alpha_2}$  that maximizes the long-term growth rate as a function of the other agents' strategy? This formulation of the problem readily gives us information on the stability of the strategies, since agents maximizing their own growth rates will converge to some equilibrium value(s) of the sharing rate  $\alpha_{ess}$  which are identified as the solutions  $\alpha^*(\alpha_{ess}) = \alpha_{ess}$ .

The first non-trivial original result of our Letter is that the value of resource sharing that maximize the growth rate  $\alpha_1^*(\alpha_2)$  for a given strategy of the other agent  $\alpha_2$  is always larger than the latter:  $\alpha_1^*(\alpha_2) > \alpha_2$  (see Fig. 1).



FIG. 1. The optimal strategy for long term growth is to cooperate more than the partner as the growth rate of individual 1 is maximized by a value of  $\alpha_1$  bigger than the partner's  $\alpha_2$ . Panel A displays the the difference between the infinite time growth rate  $g_{\alpha_1|\alpha_2}$  and the growth rate of the fully defecting case  $g_{0|0}$  as a function of  $\alpha_1$  for a fixed value  $\alpha_2 = 0.5$  and different values of the noise amplitude  $\sigma$  (colors). The analytical result of Eq. (3) (solid lines) matches the numerical simulations (markers). Red markers indicate the maxima of the curves, i.e. the value of  $\alpha_1$  maximizing agent 1's growth rate. This value lies on the right hand side of the vertical gray line that represents the values of  $\alpha_2 = 0.5$ , indicating that the optimal choice for agent 1 is to share more than agent 2. Panel B shows, for  $\sigma = 1$ , the analytical solution of long term growth rate (dark gray) and its maximum (red dots) for different values of  $\alpha_2$ . The maxima are always found for  $\alpha_1 > \alpha_2$ . This can be seen by the projection of the maxima on the horizontal plane (red solid line) that always stays above the diagonal (dashed red line,  $\alpha_1 = \alpha_2$ ). Panel c shows the dependence of the optimal  $\alpha$  line with  $\sigma$ . Decreasing the noise amplitude the curve gets closer to the diagonal while staying above it, showing that the optimal  $\alpha$  is higher than the partner's  $\alpha$  for any noise amplitude.

This mathematical result implies that, contrarily to the mechanism in the tragedy of the commons, each agent has an individual advantage in sharing *more* than the other agent. As a consequence, the evolutionary, adaptive, or learning dynamics maximizing the growth g should lead to a larger and larger level of cooperation (i.e., larger and larger values of  $\alpha$ ).

The intuition behind this result is that, in presence of fluctuations, sharing is akin to investment diversification. As already mentioned, sharing screens the agent from the detrimental effects of fluctuations. In the long-time horizon, the return from this investment (the term  $\alpha_2 \langle \exp(q_2 - q_1) \rangle_{eq}/2$ ) repays its cost (equal to  $\alpha_1/2$ ).

Robustness of the results. — One key assumption we considered so far is that the two agents experience independent fluctuations. This assumption can be easily relaxed by assuming that noises are not independent:  $\xi_i(t)\xi_i(t') = \rho\delta(t-t')$ if  $i \neq j$ . The parameter  $\rho$  quantifies the correlation between the fluctuations that two agents experience. One can expect that positive correlations ( $\rho > 0$ ) might affect our results: if agents share correlated fluctuations, the diversification effect behind the advantage of cooperation might disappear. This case is still amenable of analytical treatment and one obtains that the form of solution of Eq. (3) still applies, and the correct result is obtained by applying the substitution  $\sigma^2 \to \sigma^2(1-\rho)$  on the right hand side of the equation. Correlated fluctuation decrease the long-term growth rate because they effectively decrease the amplitude of fluctuations on which cooperation can produce a positive effect. Correlated fluctuations do not, however, change the equilibrium strategy: it is still more advantageous to cooperate more than the partner, despite the reduced relative benefit. In a similar way, increasing the group size does not alter our results. In principle, increasing the group size could have two opposite effects. On the one hand, for an individual, the relative advantage of sharing could be diminished in a larger group, as the returns of her investments are also shared with others. On the other hand, larger group sizes reduce even more the effect of fluctuations, as they correspond to an higher level of diversification. As a result, the combination of these two effects, turns out to produce even an higher advantage for cooperation. In particular, larger groups produce larger values of the growth rate, that, in the case of full cooperation converges to  $\mu - \sigma^2/(2G)$ .

Another key assumption is to describe fluctuations as white noise. In reality we might expect, e.g. in biology [18] or in economics [19], that fluctuations are time-correlated, over some timescale  $\tau$ , which could be comparable to the other timescales of the process. We introduce this effect by assuming that fluctuations have an exponentially decaying auto-correlation  $\xi_i(t)\xi_i(t') = \exp(-|t - t'|/\tau)/(2\tau)$ , which reduces to the white noise case in the limit  $\tau \to 0$ . While this case cannot be exactly solved, we approximated it using unified colored noise approximation [20]. Our analytical approximation correctly matches numerical simulation for a wide range of values of  $\tau$ . In particular, we obtain that full cooperation  $(\alpha_{ess} \to \infty)$  is not anymore an equilibrium strategy. The optimal sharing rate turns out instead to depend on the value of  $\tau$ . For any value of  $\tau$ , the equilibrium sharing rate  $\alpha_{ess}$  is a positive finite value. For small value of  $\tau$ , the equilibrium sharing rate scales as

$$\alpha_{ess} \sim \frac{\sigma\sqrt{1-\rho}}{2\sqrt{\tau}} , \qquad (4)$$

which tends to full cooperation in the white-noise limit  $\tau \to 0$ . Also, for a given value of  $\tau$ , larger levels of fluctuations and lower noise correlation produce increased cooperation.

In many settings cooperation is associated with a cost. For instance, microbes excreting enzymes in the environment, in addition to contributing to a public good, pay an additional cost for their synthesis [7, 21–23]. In our framework, a cost of cooperation can be introduced in multiple ways, which mainly differ in how the cost is defined and split among agents. In general such a term is expected to disfavour more cooperation, as it determines an additional cost at the individual level. We consider the option where the cost is proportional to the resource share rate  $\alpha_i$  by adding a term equal to  $-\alpha_i \delta x_i(t)$  to the right end side of Eq. (1). The parameter  $\delta$  represent the cost of cooperation per unit of resource shared. This model can be again solved analytically, and we obtain that for any positive value of the cooperation cost  $\delta$  it exists a finite equilibrium sharing rate. Which, in the limit of small costs, scales as

$$\alpha_{ess} \sim \frac{\sigma^2 (1-\rho)}{4\sqrt{\delta}} \ . \tag{5}$$

As expected, the level of cooperation increases with the non-shared component level of fluctuations  $\sigma^2(1-\rho)$  and we recover the case of full cooperation in the limit  $\delta \to 0$ .

Another key assumption of our framework is that we focused on infinite-time horizons. This can be relaxed by considering growth over a finite time horizons T, and evaluate the average log-returns  $\langle q_i(T) \rangle / T$ . This case is not amenable of analytical treatment and it requires relying on numerical simulations to evaluate the average log-return. It is convenient in this case to consider discrete time simulations, where agents can share a fraction  $a \in [0, 1]$  of their value at each time step. In this setting, the time-horizon correspond to a number of discrete time-steps. Fig. 2 shows that two regimes appear separated by a critical time horizon  $T^*$ . For  $T > T^*$ , the system behaves qualitatively as in the infinite time-horizon case: the individual optimizations of the log-average return lead agents to converge to a value  $a_{ess} \rightarrow 0$ . In particular, for very large time horizons we recover the prediction obtained under the diffusion limit and  $a_{ess} \rightarrow 1$ . For short time horizons ( $T < T^*$ ), defection is more advantageous than cooperation and log-return optimizations lead agents to converge to  $a_{ess} = 0$ . This result sheds light on the mechanism producing cooperation in our modeling setting: for long time-horizons, cooperation, thus investing in the other agents, continues providing returns, overcompensating its costs.



FIG. 2. The equilibrium value of the sharing resource percentage a display a phase transition as a function of the time horizon T (measured in number of discrete time steps). For short time horizons  $(T < T^*)$  it is more convenient to share less than the partner, which drive the system to a value  $a_{ess} = 0$ . Above the threshold  $T^*$  a finite value of  $a_{ess} > 0$  is evolutionary stable. For large enough values of the time horizons, one recovers the analytic predicted value of full cooperation  $(a_{ess} = 1, equivalent to <math>\alpha_{ess} \to \infty$ ) obtained in the limit  $T \to \infty$ .

Evolutionary dynamics in a finite population.—The results presented above provide a clear mathematical mechanism for the emergence and stability of cooperation in the presence of fluctuations. In order to apply these results to a more concrete example, we now focus on explicit evolutionary dynamics in a finite population. We consider a population of Nagents reproducing with non-overlapping generations at discrete time-steps. The (random) variable  $x_i$  is interpreted as fitness. Each agent i is characterized by a sharing probability  $a_i$ , which is the trait undergoing mutations and selection. Before reproduction, individuals are paired in groups of two and their fitnesses  $x_i$  are determine by the



FIG. 3. Outcomes of the evolutionary dynamics under different time-horizons. Two similar populations, with an initial a distribution with average 0.6, evolve with different time horizons  $T_{short} = 20$  (green dots) and  $T_{long} = 2000$  (blue dots). The population with short time horizon evolves towards a distribution peaked in a = 0 while the long time horizon one, in the opposite direction, towards a a = 1.

discrete multiplicative process with sharing, run over a finite time-horizon T with initial condition  $x_i(0) = 1$ . The fitness of each individual is therefore a stochastic variable that depends on the values of  $a_i$  of both individuals in the pair. After this step, the pairs are broken up and each individual reproduces proportionally to her fitness value  $x_i$ .

As expected from previous results of population genetics in fluctuating environments [12], evolution drives the population to traits that maximize the expected log-fitness. Fig. 3 shows the population average values of resource sharing probability a over time. For a short time horizon T, defection dominates and the distribution of a is peaked close to 0, with some variance given by mutations and genetic drift. Conversely, when the time horizon is large enough, the vast majority of individuals cooperate, and a peaks close to one.

Discussion. — In this Letter, we have discussed the optimal sharing strategy of agents in presence of multiplicative stochastic growth. Cooperation can lead to faster growth of individual agents, therefore becoming an evolutionary stable strategy. In this context, cooperation effectively screens agents from the detrimental effect of fluctuations: by cooperating, an agents effectively diversify its investment, producing an higher return in the long term. This is a sustainable strategy only if agents act with a long-time horizon and this altruistic investment have the time to repay off. For short time-horizons, defection becomes again the evolutionary stable strategy.

Our approach differs considerably from previously identified mechanisms that explain the emergence and the stability of cooperation [8]. Our approach does not in fact invoke multilevel selection (like in group selection or kin selection), as we consider individuals that only maximize their own growth rate. Moreover, direct reciprocity [24] is not the ingredient determining cooperation in our framework. Direct reciprocity requires agents to change their actions based on the previous actions of other agents. In our setting, given a value of the resource sharing rate  $\alpha_2$ of agent 2, even if agent 1 is allowed to choose her own sharing rate  $\alpha_1$  once for all, her optimal choice would be to share more than the other ( $\alpha_1 > \alpha_2$ ).

More generally, mapping our results in classic evolutionary game theory requires care. One could be tempted to consider the process described in Eq. (1) analogous to a repeated public good game, where the single game iteration correspond to a discrete step of the growth and sharing process and the value of  $\dot{x}$  would be the payoff. This mapping is indeed not straightforward as in classic game theory payoffs do not accumulate in a multiplicative fashion, and, more importantly, do not depend on the previous history of iterations. Note however, that considering  $\dot{q}$  as the single iteration payoff would not solve the problem, as also  $\dot{q}$  depends on the previous history of the game (i.e. on time through x(t)). A more precise way to connect our results to game theory would be to interpret the fully history defined by Eq. (1) as a single iteration of a game. In this context,  $g_{\alpha_1|\alpha_2}$  is — by definition — the payoff of the game, and  $\alpha_{ess}$  its unique Nash equilibrium and evolutionary stable strategy.

Our framework can therefore be seen as a mechanistic way to define a payoff matrix, in the context of agents trying to maximise their future wealth. The fundamental origin of the advantage of cooperation in our framework is due to the non-ergodicity of stochastic exponential growth, which effectively determines an individual advantage in reducing the level of fluctuations. Increasing the rate of cooperation comes at an immediate individual cost, as part of the wealth is diluted among agents as a public good, and has a long-term return, as the wealth shared with others is subject to independent fluctuations. The surprising result of this Letter is that the second effect is stronger than the first one, making it more advantageus — at the individual level — to cooperate.

The application to a concrete setting in population and evolutionary biology is not straightforward. In particular, there are two possible ways to interpret the meaning of the variable x. One possibility would be to interpret it as a (sub-)population abundance growing exponentially in presence of environmental fluctuations. The caveat in this interpretation is that it requires to assume that the effect of the public good on the population growth rate is independent of the abundance. Another possibility would be to interpret x as an internal variable related to fitness (as we did in our evolutionary model), which would require to identify such a individual variable characterized by the dynamics of Eq. (1).

It would be interesting to extend our framework in multiple direction. First of all, we assume that individuals share their value, but the scenario where they share only the income is potentially very interesting. We also assume that the group size is fixed, but for many biological (e.g., origin of multicellularity [25, 26]) and sociological (e.g., group formation [27]) it would be interesting to treat it as a dynamical variable that can be optimised.

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## S1. DIFFUSION APPROXIMATION FOR RANDOM EXPONENTIAL GROWTH

Let us consider the stochastic exponential growth in discrete time

$$x(t+1) = x(t)\zeta(t) \tag{S6}$$

where  $\zeta(t)$  is a positive random variable from an arbitrary distribution  $\rho(\zeta)$ . In particular, we define its mean  $\langle \zeta \rangle = 1 + m$  and variance  $\langle \zeta^2 \rangle - \langle \zeta \rangle^2 = s^2$ . We can rewrite the previous equation as

$$x(t+1) = x(t) + mx(t) + sx(t)\nu(t) , \qquad (S7)$$

where  $\nu(t)$  is a random variable with mean 0, variance 1 and arbitrary distribution (trivially related to the one of  $\zeta$ . Without loss of generality, we can change the time units and define  $\Delta t$  as the time between two steps of the process

$$x(t + \Delta t) = x(t) + mx(t) + sx(t)\nu(t)$$
(S8)

It is convenient to define  $\mu = m/\Delta t$  and  $\sigma^2 = s^2/\Delta t$ , from which we obtain

$$x(t + \Delta t) = x(t) + \mu \Delta t x(t) + \sigma \sqrt{\Delta t} x(t) \nu(t) .$$
(S9)

We are interested in timescales much longer than the discrete time-steps. It is therefore convenient to consider the limit  $\Delta t \rightarrow 0$ . As well known [28], the trajectories of Eq.(S9) converge, in the limit  $\Delta t \rightarrow 0$  to

$$\dot{x} = \mu x + \sigma x \xi(t) , \qquad (S10)$$

where  $\xi(t)$  is a white Gaussian noise.

## S2. DIFFUSION APPROXIMATION FOR RANDOM EXPONENTIAL GROWTH WITH SHARING

In the following we consider the diffusion approximation for different models involving sharing between agents. For simplicity we restrict the analysis to the case of two agents.

### S2.1. Growth, then sharing total wealth

We consider a model divided in two steps. In the first intermediate step, the wealth of each agent grows by a random factor. In the second step, the resulting wealth are pooled and shared among agents. We describe the first intermediate step is

$$x_1(t+1/2) = x_1(t)\zeta_1(t)$$
  

$$x_2(t+1/2) = x_2(t)\zeta_2(t)$$
(S11)

The second intermediate step involves the sharing of the wealth. A fraction  $a_i$  of the wealth of each agent is shared in a pool and then equally divided among agents

$$x_1(t+1) = x_1(t+1/2)(1-a_1) + \frac{a_1x_1(t+1/2) + a_2x_2(t+1/2)}{2}$$
  

$$x_2(t+1) = x_2(t+1/2)(1-a_2) + \frac{a_1x_1(t+1/2) + a_2x_2(t+1/2)}{2},$$
(S12)

Putting the two steps together, one obtains

$$x_{1}(t+1) = x_{1}(t)\zeta_{1}(t) + \frac{a_{2}x_{2}(t)\zeta_{2}(t) - a_{1}x_{1}(t)\zeta_{1}(t)}{2}$$

$$x_{2}(t+1) = x_{2}(t)\zeta_{2}(t) + \frac{a_{1}x_{1}(t)\zeta_{1}(t) - a_{2}x_{2}(t)\zeta_{2}(t)}{2}.$$
(S13)

Following the same procedure used in section S1, we introduce the discrete time step duration  $\Delta t$ , and the parameters  $\mu = m/\Delta t$  and  $s^2 = \sigma^2/\Delta t$ . We also introduce the sharing rate  $\alpha = a/\Delta t$ . We obtain

$$x_1(t+\Delta t) = x_1(t)(1+\mu\Delta t + \sigma\sqrt{\Delta t}\nu_1(t)) + \Delta t \frac{\alpha_2 x_2(t)(1+\mu\Delta t + \sigma\sqrt{\Delta t}\nu_2(t)) - \alpha_1 x_1(t)(1+\mu\Delta t + \sigma\sqrt{\Delta t}\nu_1(t))}{2},$$
(S14)

and an analogous equation for  $x_2$ . In the limit  $\Delta t \to 0$ , this equation converges to

$$\dot{x_1} = \mu x_1 + \sigma x_1 \xi_1(t) + \frac{\alpha_2 x_2 - \alpha_1 x_1}{2} , \qquad (S15)$$

where  $\xi_1(t)$  is a Gaussian white noise, with  $\langle \xi_1(t)\xi_2(t')\rangle = \rho\delta(t-t')$ , where  $\rho$  is the correlation coefficient between  $\zeta_1(t)$  and  $\zeta_2(t)$ 

$$\rho = \frac{\langle \zeta_1(t)\zeta_2(t)\rangle - m^2}{s^2} , \qquad (S16)$$

#### S2.2. Share total wealth, then grow

This model differs from the previous just because the order of the steps is reverted. The first step is

$$x_1(t+1/2) = x_1(t) + \frac{a_2 x_2(t) - a_1 x_1(t)}{2} , \qquad (S17)$$

while the second intermediate step is

$$x_1(t+1) = x_1(t+1/2)\zeta_1(t) , \qquad (S18)$$

and equivalent for the other agent. Taken together, we obtain

$$x_1(t+1) = x_1(t)\zeta_1(t) + \frac{a_2x_2(t) - a_1x_1(t)}{2}\zeta_1(t) .$$
(S19)

It is easy to see that the same considerations that apply to the previous model apply also to this one, and one obtains exactly the same diffusion limit, with the same interpretation of parameters.

### S3. SOLUTION OF THE DIFFUSION LIMIT

We consider the diffusion limit with two agents

$$\dot{x}_1 = \mu x_1 + x_1 \sigma \xi_1(t) + \frac{\alpha_2 x_2 - \alpha_1 x_1}{2}$$

$$\dot{x}_2 = \mu x_2 + x_2 \sigma \xi_2(t) + \frac{\alpha_1 x_1 - \alpha_2 x_2}{2}$$
(S20)

where, in full generality, the noise can be correlated both among individuals and in time

$$\langle \xi_i(t)\xi_j(t')\rangle = (\rho(1-\delta_{ij})+\delta_{ij})\frac{e^{-\frac{|t-t'|}{\tau}}}{2\tau} .$$
(S21)

The limit  $\tau \to 0$  corresponds to the  $\delta$ -correlate case, where  $\langle \xi_i(t)\xi_i(t')\rangle = \delta(t-t')$ 

By introducing  $q_i := log(x_i)$ , we obtain

$$\dot{q}_{1} = \mu - \frac{\sigma^{2}}{2} + \sigma\epsilon_{1} + \frac{\alpha_{2}e^{q_{2}-q_{1}} - \alpha_{1}}{2}$$

$$\dot{q}_{2} = \mu - \frac{\sigma^{2}}{2} + \sigma\epsilon_{2} + \frac{\alpha_{1}e^{q_{1}-q_{2}} - \alpha_{2}}{2} ,$$
(S22)

where the term  $-\sigma^2/2$  appears because of Itô integration rules. It is useful to write an equation for the difference of log-returns  $d := q_2 - q_1$ , which reads

$$\dot{d} = \frac{\alpha_1 e^{-d} - \alpha_2 e^d}{2} + \frac{\alpha_1 - \alpha_2}{2} + \sigma \sqrt{2(1-\rho)}\epsilon(t)$$
(S23)

where the noise term  $\epsilon(t)$ , with covariance,

$$\langle \epsilon(t)\epsilon(t')\rangle = \frac{e^{\frac{|t-t'|}{\tau}}}{2\tau}$$
(S24)

was obtained using  $\xi_1(t) - \xi_2(t) = \sqrt{2(1-\rho)}\epsilon(t)$ . The average log-return

$$g_{\alpha_1|\alpha_2} := \lim_{t \to \infty} \frac{\langle q_1 \rangle}{t} \tag{S25}$$

can be calculated from Eq. (S22) by solving

$$\langle \dot{q}_1 \rangle = \mu - \frac{\sigma^2}{2} + \frac{\alpha_2 \langle e^d \rangle - \alpha_1}{2} , \qquad (S26)$$

which requires to calculate  $\langle e^d \rangle$  using Eq. (S41). Since Eq. (S41) admits a stationary distribution  $P^*(d)$ , for large times  $\langle e^d \rangle$  will converge to the time independent quantity

$$\langle e^d \rangle_* := \int_{-\infty}^{\infty} \mathrm{d}d \ P^*(d) e^d , \qquad (S27)$$

and therefore we obtain the solution

$$g_{\alpha_1|\alpha_2} = \mu - \frac{\sigma^2}{2} + \frac{\alpha_2 \langle e^d \rangle_* - \alpha_1}{2} \tag{S28}$$

Given a value of  $\alpha_2$ , we can ask what is the value of  $\alpha_1$  that maximizes the growth rate of agent 1

$$\alpha_*(\alpha_2) := \arg\max_{\beta} g_{\beta|\alpha_2} \ . \tag{S29}$$

We can imagine an iterative process where each agent, in turn, sets her own sharing rate to the optimal value. If  $\alpha_1^*(\alpha_2) > \alpha_2$ , the first agent will set her sharing rate  $\alpha_1$  to a value larger than the one of agent 2. Then agent 2, will set her own sharing rate to the value that maximize her growth rate given a sharing rate of the other agent equal to the new value  $\alpha_1$ . If we define the function  $\alpha^*(\alpha) = \arg \max_{\beta} g_{\beta|\alpha}$ , we expect that, in this iterative process,  $\alpha$  will increase when  $\alpha^*(\alpha) > \alpha$  and decrease when  $\alpha^*(\alpha) < \alpha$ . We define the equilibrium value(s) of the sharing rate the solution of  $\alpha^*(\alpha_{ess}) = \alpha_{ess}$ . If  $\alpha^*(\alpha) > \alpha$ , we will say that  $\alpha_{ess} = \infty$  (which correspond to full cooperation), while if  $\alpha^*(\alpha) < \alpha$ , then  $\alpha_{ess} = 0$  (full defection). Figure S4 shows the values of  $\alpha^*(\alpha)$  under different modeling scenarios.

It is important to notice that  $g_{\alpha_1|\alpha_2} = g_{\alpha_2|\alpha_1}$ . This can be shown by noticing that

$$g_{\alpha_2|\alpha_1} = \lim_{t \to \infty} \frac{\langle q_2 \rangle}{t} = \lim_{t \to \infty} \frac{\langle q_1 \rangle + \langle d \rangle}{t} = g_{\alpha_1|\alpha_2} + \lim_{t \to \infty} \frac{\langle d \rangle}{t} .$$
(S30)

Since d admits a stationary distribution, for large times  $\langle d \rangle$  converges to a finite value  $\langle d \rangle_*$  and therefore the limit on the right end equals zero.

Notice that since  $g_{\alpha_2|\alpha_1}$  is symmetric, whenever we find an solution of  $\alpha^*(\alpha_{ess}) = \alpha_{ess}$ , the pure strategies  $\alpha_1 = \alpha_{ess}$  and  $\alpha_2 = \alpha_{ess}$  are (at least weak) Nash equilibrium. By construction  $\alpha_{ess} = \arg \max_{\alpha_1} g_{\alpha_1|\alpha_{ess}}$  implies that  $\alpha_1 = \alpha_{ess}$  is a best response to  $\alpha_2 = \alpha_{ess}$ . By symmetry it holds that also  $\alpha_{ess} = \arg \max_{\alpha_2} g_{\alpha_2|\alpha_{ess}}$ , so the converse is also true.

### **S3.1.** Delta-correlated noise: $\tau = 0$

If no time correlation is present in the multiplicative noises  $(\tau = 0)$  the stationary distribution of d can be calculated exactly. Defining  $f(x) := \frac{\alpha_1 e^{-x} - \alpha_2 e^x}{2} + \frac{\alpha_1 - \alpha_2}{2}$  we have



FIG. S4. Value of the sharing rate  $\alpha^*(\alpha)$  that maximizes an agent long term growth rate as a function of the sharing rate of the partner  $\alpha$ . Panel A shows the curves obtained for different values of the noise autocorrelation time  $\tau$  (colors) for  $\rho = 0$  and cooperation cost  $\delta = 0$ . Panel B shows the optimal value of the sharing rate as function of the partner sharing rate for different values of  $\rho$  (colors) for  $\tau = 0$  and  $\delta = 0$ . Panel C shows the optimal value of the sharing rate as function of the partner sharing rate for different values of the cooperation cost  $\delta$  (colors) for  $\tau = 0$  and  $\rho = 0$ . The dashed line represent the value of the sharing rate equal to the one of the partner. When the optimal sharing rate exceeds the value of the sharing rate of the sharing rate is expected to decrease (arrows pointing to the left). In Panel A and C, the curves of the optimal value of the sharing rate intercept the dashed line at finite value of the partner  $\alpha = \alpha^*(\alpha)$ , which correspond to the equilibrium value  $\alpha_{ess}$ . In panel B, the curves are always above the dashed line, implying the  $\alpha_{ess} \to \infty$ .

$$P^*(d) \propto \exp\left(\frac{1}{\sigma^2(1-\rho)} \int_0^d \mathrm{d}x f(x)\right) , \qquad (S31)$$

from which we obtain

$$P^{*}(d) = \frac{1}{\mathscr{Z}} \exp\left(\frac{(1 - e^{-d})(\alpha_{1} - e^{d}\alpha_{2}) + (\alpha_{1} - \alpha_{2})d}{2\sigma^{2}(1 - \rho)}\right) ,$$
(S32)

where

$$\mathscr{Z} := \int_{-\infty}^{\infty} \mathrm{d}d \, \exp\left(\frac{(1-e^{-d})(\alpha_1 - e^d\alpha_2) + (\alpha_1 - \alpha_2)d}{2\sigma^2(1-\rho)}\right) = \\ = 2e^{-\frac{\alpha_1 + \alpha_2}{2(\rho-1)\sigma^2}} \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1 - \alpha_2}{4(\rho-1)\sigma^2}} K_{\frac{\alpha_2 - \alpha_1}{2(1-\rho)\sigma^2}} \left(\frac{\sqrt{\alpha_1\alpha_2}}{(1-\rho)\sigma^2}\right) ,$$
(S33)

where  $K_{\beta}(z)$  is the modified Bessel function of the second kind. From the previous expression we obtain

$$\langle e^d \rangle_* = \frac{1}{\mathscr{Z}} \int_{-\infty}^{\infty} \mathrm{d}d \ e^d \exp\left(\frac{(1-e^{-d})(\alpha_1 - e^d\alpha_2) + (\alpha_1 - \alpha_2)d}{2\sigma^2(1-\rho)}\right) = \frac{\sqrt{\frac{\alpha_1}{\alpha_2}}K_{-1+\frac{\alpha_2-\alpha_1}{2(1-\rho)\sigma^2}}\left(\frac{\sqrt{\alpha_1\alpha_2}}{(1-\rho)\sigma^2}\right)}{K_{\frac{\alpha_2-\alpha_1}{2(1-\rho)\sigma^2}}\left(\frac{\sqrt{\alpha_1\alpha_2}}{(1-\rho)\sigma^2}\right)},$$
(S34)

which leads to

$$g_{\alpha_{1}|\alpha_{2}} := \mu - \frac{\sigma^{2}}{2} - \frac{\alpha_{1}}{2} + \frac{\sqrt{\alpha_{1}\alpha_{2}}K_{-1 + \frac{\alpha_{2} - \alpha_{1}}{2(1 - \rho)\sigma^{2}}}\left(\frac{\sqrt{\alpha_{1}\alpha_{2}}}{(1 - \rho)\sigma^{2}}\right)}{2K_{\frac{\alpha_{2} - \alpha_{1}}{2(1 - \rho)\sigma^{2}}}\left(\frac{\sqrt{\alpha_{1}\alpha_{2}}}{(1 - \rho)\sigma^{2}}\right)} .$$
(S35)

If the level of cooperation is the same for the two agents ( $\alpha_1 = \alpha_2 = \alpha$ ), Eq. (S35) reduces to

$$g_{\alpha|\alpha} := \mu - \frac{\sigma^2}{2} + \frac{\alpha}{2} \left( \frac{K_{-1}\left(\frac{\alpha}{(1-\rho)\sigma^2}\right)}{K_0\left(\frac{\alpha}{(1-\rho)\sigma^2}\right)} - 1 \right) , \qquad (S36)$$



FIG. S5. Sharing with a group of individuals instead of a unique partner does not affect the stability of cooperation. The relative long term growth rate represents the difference between the typical long term growth rate of individual 1 and the growth rate in the fully defective case  $\alpha_i = 0 \quad \forall i$ . We show it in function of how much agent 1 shares ( $\alpha_1$ ) keeping fixed the amount shared by other agents in the group  $\alpha_i = 0.5 \quad \forall i \neq 1$ . The latter function presents maxima (red markers) for  $\alpha_1 > \alpha_i$  independently from the group size G, implying an optimal  $\alpha$  higher to the  $\alpha$  of the other agents in the group. All the simulations were obtained with  $\mu = 0.5$  and  $\sigma = 1$ .

which is a strictly increasing function of  $\alpha$ . In the case of full cooperation (i.e. in the limit  $\alpha \to \infty$ ), one obtains  $g_{\infty|\infty} := \mu - (1+\rho)\sigma^2/4$ .

The analytical results are valid for two agents. Figure S5 shows that the same phenomenology applies for groups of arbitrary sizes.

#### **S3.2.** The $\tau \neq 0$ case

If the noise is time correlated we can use the Unified Coloured Noise Approximation [20] to find the stationary probability distribution of d, which reads

$$P^{*}(d) = \frac{|1 - \tau f'(d)|}{\mathscr{Z}} \exp\left(\frac{1}{\sigma^{2}(1 - \rho)} \left(-\frac{\tau}{2}f(d)^{2} + \int_{0}^{d} \mathrm{d}x f(x)\right)\right) ,$$
(S37)

where  $f(x) := \frac{\alpha_1 e^{-x} - \alpha_2 e^x}{2} + \frac{\alpha_1 - \alpha_2}{2}$ 

Fig. S6 shows that the analytical solution is in agreement with the numerical simulations for all the different parameter combination examined. For small values of  $\tau$  the equilibrium sharing rates scales as

$$\alpha_{ess} = \frac{\sigma(1-\rho)}{2} \frac{1}{\sqrt{\tau}} . \tag{S38}$$

The growth rate at the equilibrium value scales as

$$g_{\alpha_{ess}|\alpha_{ess}} = \frac{\sigma^2}{4} \left( 1 + \rho - \sigma \sqrt{1 - \rho} \sqrt{\tau} \right) .$$
(S39)

Fig. S7 shows the value of the equilibrium sharing rate  $\alpha_{ess}$  as a function of the noise autocorrelation timescale  $\tau$ . Increasing  $\tau$  always leads to a reduction of the equilibrium sharing rate and the corresponding growth rate.

## **S4. DISCRETE-TIME NUMERICAL SIMULATIONS**

In the discrete time simulations each agent had a value  $x_i(t)$ . At every step the value  $x_i$  was multiplied by a random factor  $\chi$  with probability 1/2 or 1/ $\chi$  with probability 1/2. To obtain Fig. 2 we used  $\chi = 2$ . Each agent then shared a fraction  $a_i$  of her total wealth in a common pool, which was then equally distributed among all agents. We always considered  $x_i(0) = 1$  for all the agents.



FIG. S6. The typical long term growth rate of individual 1 as a function of how much he shares ( $\alpha_1$ ). Simulation (markers) are in accordance with the analytical calculations (solid lines). Each column of panels is calculated for a different value of the partner's share  $\alpha_2$ . **A**,**B**,**C**: show the behaviour at different values of  $\tau$ , the decorrelation time of the multiplicative noise (the lighter the higher  $\tau$ ). In red a marker representing the maximum of each curve. We can see that, as  $\tau$  increases the best choice of  $\alpha_1$  passes from being grater than  $\alpha_2$  to the opposite. **D**,**E**,**F**: show the behaviour at different values of  $\rho$ , the correlation between the multiplicative noise of the two individuals (the lighter the higher). In red a marker representing the maximum of each curve. We can see that, as  $\rho$  increases the best choice of  $\alpha_1$  always remains greater than  $\alpha_2$ .

We estimated the growth rate  $g_{a_1|a_2}$  over a time horizon T as the average of the logarithm of  $x_1(T, a_1, a_2)/T$  over  $10^5$  simulations. For low values of T, below the critical  $T^*$ , we have  $g_{a+\delta a|a}(T) < g_{a|a}(T)$  for every a, making full defection stable. Above the critical time horizon  $T^*$ , we obtain that  $g_{a+\delta a|a}(T) > g_{a|a}(T)$  for  $a < a_{ess}(T)$  and vice versa.

To obtain the equilibrium sharing probability  $a_{ess}$  at a given time horizon T we considered the difference in the typical growth of an an agent cooperating as much as the partner  $a_1 = a_2$  and one cooperating more than the partner  $a_1 = a_2 + \delta a$ , with  $\delta a = 0.025$ . We define  $\Delta(a) = (g_{a+\delta a|a}(T) - g_{a|a}(T))/\delta a$  as the difference between the growth rates (which, in the limit  $\delta a \to 0$  approximates the derivative of the growth rate). We estimated numerically the equilibrium value of the sharing fraction as the zero of the function f(a) obtained by fitting  $\Delta(a)$  with a linear function.

## S5. EVOLUTIONARY SIMULATIONS

We simulated the evolution of N = 100 individuals using the Wright-Fisher model. Each individual was characterized by a cooperation parameter  $a_i$  initially uniformly distributed in [0.3, 0.7].

At each evolutionary time step, corresponding to one generation, individuals were grouped in pairs. Each agent was associated with a value  $f_i(t)$ , which changed over time accordingly to a discrete stochastic growth and share, with  $\chi = 2.5$ . All the individuals were initialized with a value  $f_i(0) = 1$ .

After each iteration individuals reproduced proportionally to the final value of the fitness  $f_i(T)$ , obtained by running the discrete growth and share model for T discrete steps.

The offspring generated have a share parameter  $a_i$  equal to the one of the parent plus a random mutation normally distributed with amplitude 0.05 (unless the resulting value was negative or larger than 1, in which case was set to 0



FIG. S7. Equilibrium sharing rate  $\alpha_{ess}$  (panel A) and long-term growth rate (panel B)  $g_{\alpha_{ess}|\alpha_{ess}}$  as a function of the noise autocorrelation time. All the panels were obtained from a numerical integration of Eq. (S37), using  $\mu = 1$ ,  $\sigma = \sqrt{2}$  and  $\rho = 0$ . The dashed lines refer to the scaling behavior of Eq. (S38) and Eq. (S39)

or 1, respectively).

Following reproduction, the new individuals were grouped in pairs randomly, and a new process started, resulting in a new value  $f_i(T)$ .

We consider two values of T, corresponding to a short time horizon (T = 20) and a long one (T = 2000).

### S6. COSTLY COOPERATION

In this section we consider the case of costly cooperation. A cost can be introduced in multiple ways. The main different between alternatives is whether the cost is shared by the group (e.g., if a fraction of the public good is dissipated and lost) or only affects the cooperatios. We consider latter, being a more extreme case, where a cooperator pays a direct cost  $\delta$ , proportional to the level of cooperation

$$\dot{x}_i(t) = \mu x_i(t) + \sigma x_i(t)\xi_i(t) - \alpha_i \delta x_i + \frac{1}{G} \sum_{j \neq i} \left( \alpha_j x_j(t) - \alpha_i x_i(t) \right) , \qquad (S40)$$

Following the same procedure of the case without cooperation cost ( $\delta = 0$ ) we obtain, in the case  $\tau = 0$ 

$$\dot{d} = \frac{\alpha_1 e^{-d} - \alpha_2 e^d}{2} + (1 + 2\delta) \frac{\alpha_1 - \alpha_2}{2} + \sigma \sqrt{2(1 - \rho)} \epsilon(t)$$
(S41)

from which we obtain

$$g_{\alpha_1|\alpha_2} := \mu - \frac{\sigma^2}{2} - \frac{\alpha_1}{2} (1+2\delta) + \frac{\sqrt{\alpha_1 \alpha_2} K_{-1+(1+2\delta) \frac{\alpha_2 - \alpha_1}{2(1-\rho)\sigma^2}} \left(\frac{\sqrt{\alpha_1 \alpha_2}}{(1-\rho)\sigma^2}\right)}{2K_{(1+2\delta) \frac{\alpha_2 - \alpha_1}{2(1-\rho)\sigma^2}} \left(\frac{\sqrt{\alpha_1 \alpha_2}}{(1-\rho)\sigma^2}\right)}$$
(S42)

Fig. S8 shows the equilibrium sharing rate for different values of  $\delta$ .

In the case of agents with equal strategy, we obtain

$$g_{\alpha|\alpha} := \mu - \frac{\sigma^2}{2} + \frac{\alpha}{2} \left( \frac{K_{-1}\left(\frac{\alpha}{(1-\rho)\sigma^2}\right)}{K_0\left(\frac{\alpha}{(1-\rho)\sigma^2}\right)} - 1 - 2\delta \right) , \qquad (S43)$$

which has a maximum at finite values of  $\alpha$ .

If we take the derivative over  $\alpha$ , expand for large values of  $\alpha$  we obtain

$$\frac{\partial}{\partial \alpha} \left( \alpha \frac{K_{-1} \left( \frac{\alpha}{(1-\rho)\sigma^2} \right)}{K_0 \left( \frac{\alpha}{(1-\rho)\sigma^2} \right)} - \alpha - 2\delta \alpha \right) \approx -2\delta + \frac{(1-\rho)^2 \sigma^4}{8\alpha^2} + O\left(\epsilon^{5/2}\right)$$
(S44)



FIG. S8. equilibrium sharing rate  $\alpha_{ess}$  and long-term growth rate  $g_{\alpha_{ess}|\alpha_{ess}}$  as a function of the cooperation cost  $\delta$ . The black lines correspond are obtained via a numerical maximization of Eq. (S43). The gray dashed line in panel A corresponds to the approximate solution of Eq. (S45). The gray dashed line in panel B corresponds instead to Eq. (S46). As expected, the approximation correctly matches the trend for small values of  $\delta$ . All the panels were obtained using  $\mu = 1$ ,  $\sigma = \sqrt{2}$ , and  $\rho = 0$ .

and set to zero we obtain

$$\alpha_{ess} \sim \frac{(1-\rho)\sigma^2}{4\sqrt{\delta}} \tag{S45}$$

and the growth rate simply reduces to

$$g_{\alpha_{ess}|\alpha_{ess}} = \mu - \frac{1}{4}\sigma^2 \left( 1 + \rho + \sqrt{\delta}(2 - \rho(2 - \rho)) \right)$$
(S46)

Fig. S8 shows that this approximation correctly matches the trend of  $\alpha_{ess}$  and  $g_{\alpha_{ess}|\alpha_{ess}}$  for small values of  $\delta$ .